Cross-Region, Cross-Sector Asset Allocation with Regimes

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Abstract

Cross-region and cross-sector asset allocation decisions are one of the most fundamental issues in international equity portfolio management. Equity returns exhibit higher volatilities and correlations, and lower expected returns, in bear markets compared to bull markets. However, static mean-variance analysis fails to capture this salient feature of equity returns. Using a regime switching model across both regions and sectors, the regime-dependent asset allocation substantially outperforms the static mean-variance allocation. This outperformance is robust both in-sample and out-of-sample, as well as under various asset allocation constraints. In addition, optimal allocation across sectors provide greater benefits compared to international diversification, which is characterized by higher returns, lower risks, lower correlations with the world market and a higher Sharpe ratio..

Keywords: strategic asset allocation, sector rotation, regime switching, Markov switching

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1 Introduction

For international equity fund managers, the asset allocation decision – how much to invest in each major region and sector – is a key determinant of their portfolio performance. Region and sector weights can only be determined on the basis of a model that characterizes the joint distribution of equity returns. Most studies assume that equity returns are generated by a stationary process with mean, variances and covariances of returns that do not vary over time. However, there is growing empirical evidence that equity returns follow a rather more complicated process with multiple 'regimes', namely different market or economic conditions, each of which is associated with a very different distribution of returns (Garcia and Perron (1996), Perez-Quiros and Timmermann (2000), Longin and Solnik (2001), Ang and Bekaert (2002a and 2002b), Ang and Chen (2002), Ang and Bekaert (2004), Guidolin and Timmermann (2007)). This empirical feature of equity returns highlights the need for a dynamic asset allocation model that accounts for different distributions of asset returns in different regimes.

Our paper investigates whether a dynamic asset allocation that accounts for regimes can allocate assets more efficiently across regions and sectors, that is, generate higher returns for investors, than a static Markowitz' (1952) mean-variance analysis. Similar to Hamilton's (1989) regime switching model, we define regimes as unobservable to investors, who can only infer regime probabilities from past return observations. We consider two regimes where the first (second) regime has the characteristics of a bull (bear) market with high (low) returns and low (high) volatilities. These two regimes offer very different investment opportunities so investors' asset allocations vary significantly over time as they revise their beliefs about the underlying regime probabilities.

We demonstrate that a regime-dependent asset allocation that holds different mean-variance efficient portfolios in different regimes outperforms the single period mean-variance optimal portfolio. This outperformance is observed in both cross-region and cross-sector allocations. Most evidently, sector allocations provide higher diversification benefits and perform better than optimal allocation across regions.

Our study is most closely related to Ang and Bekaert (2004) who document how the presence of regimes can be incorporated into two asset allocation programs - a global asset allocation setting with six equity markets and a market timing setting for U.S. cash, bonds and equity. Ang and Bekaert (2004) find that regime switching strategies have the potential to outperform the static mean-variance analysis because they are able to capture different distributions of asset returns at different times in the business cycle. However, their results indicate that the superior performance of regime switching strategies may be linked to a specific historical period, in their case 1975 to 2000. While our study confirms the Ang and Bekaert (2004) results, we also find that the regime dependent portfolio outperforms the static strategy in an extended time period (2000 to 2010).

We extend the research of Ang and Bekaert (2004) and contribute to the asset allocation literature by demonstrating how the presence of regimes is exploitable in a sector allocation program. Previous research that investigates the merits of specific sector rotation strategies is surprisingly scarce given that the industry level allocation is a fundamental component of most portfolio constructions. Conventional market wisdom also posits that different sectors perform differently in various stages of the business cycle (or financial conditions). The focus of previous sector rotation studies has been on selecting appropriate indicators that can identify when the portfolio should be shifted to a more defensive or aggressive position. Sassetti and Tani (2006) find that the sector rotation strategy is profitable using a number of technical indicators including relative strength and moving averages. However, these techniques are heuristic rather than scientific in nature and lack theoretical justification. Conover et al. (2008) document a US sector rotation strategy that out-performs the market by 3.40% per year, using the Federal Reserve monetary policy as an indicator. Nevertheless, it is difficult to generalize these results to settings outside the US. Both Sassetti and Tani (2006) and Conover et al. (2008) also fail to consider the optimal asset allocation implications under different financial or economic conditions. Our paper addresses these issues using a regime switching model that derives the regime shift indicators statistically from the stock returns data and returns and volatilities in different regimes, hence allowing investors to strategically invest in the optimal defensive (aggressive) portfolio in bear (bull) markets.

The empirical results with respect to our regime switching sector rotation strategy are important. In our out-of-sample analysis from 2005 to 2010, the regime-dependent sector allocation delivers an average annual return of 14.47% (Sharpe ratio = 0.88), compared to the annual returns on a static mean-variance sector allocation and the world market portfolio of 6.93% (Sharpe ratio = 0.39) and 1.59% (Sharpe ratio = -0.07) respectively.

Our paper also contributes to the ongoing debate of whether international diversification is more important and beneficial than allocation across sectors. Solnik (1974) demonstrates that diversification across countries provides greater risk reduction than diversification across industries. Heston and Rouwenhorst (1995) and Griffin and Karolyi (1998) find that the benefits of international diversification stem largely from geographical diversification, rather than from industrial diversification. Conversely, Cavaglia et al. (2000), Baca et al. (2000) and Cavaglia and Moroz (2002) find evidence that industry factors have grown in importance in recent years. Brooks and Catao (2000) further show that industry sectors are becoming more important in explaining portfolio risk than the country factor since 1995. Research by Brooks and Del Negro (2004) articulates that the rise in industry effects is simply a temporary phenomenon associated with the information technology bubble, rather than a reflection of greater economic integration across countries. Ferreira and Gama (2004) document that, in the 1990s, correlations among local industries have declined and there is more of a penalty for not being well diversified across industries.

Most of these earlier studies use factor approaches and do not directly consider the optimal asset allocation implications. Our optimal asset allocation results support the view of an increasingly important industry diversification effect. We document that during the period from 1995 to 2009, cross-sector optimal allocation generates higher returns, lower risk, lower correlations with the world market, and higher Sharpe ratios than cross-region optimal allocation. In addition, a regime dependent sector allocation outperforms the regime dependent regional allocation substantially.

This paper is organized as follows. Section 2 discusses the previous literature and section 3 describes our data. Section 4 documents the regime switching model and the model estimation. Section 5 illustrates the asset allocation methodology and compares the performance of regime-dependent portfolios against the static mean-variance optimal portfolios. Section 6 concludes the paper.

2 Literature

2.1 Strategic asset allocation and regime switching

Markowitz' (1952) mean-variance analysis, as a foundational approach to understanding the relation of risk and return, has been widely adopted as a central paradigm in developing long-term asset allocation parameters. However the Markowitz model ignores several important factors. Most importantly, the analysis is static, which unrealistically assumes that investors only care about risks to wealth that are one period ahead. Financial economists have long realized since the work of Samuelson (1969) and Merton (1969, 1971, 1973) that the solution to a multi-period portfolio choice problem can be very different from the solution to a static portfolio choice problem. This is indeed significant, because estimation of the long-run risk and return relationship is critically important to an investors' ability to achieve long-term objectives. Mean-variance optimization of a multi-period portfolio has been developed in studies of "strategic asset allocation" by Brennan et al. (1997) as a method of guiding investor decision-making over the business cycle.

One particular approach to addressing strategic asset allocation is to characterize investment opportunity sets into different regimes (or periods of time), and model the process and probabilities of regime switches over time. Hamilton (1989) is the first study that uses a regime shifting framework to model switches between periods of high and low volatility of asset returns. In a regime shifting framework, the transition from one regime to another follows a Markov chain. That is, at each point in time, there is a certain probability that the process will stay in the same regime, or transition to another regime in the next period. These transition probabilities might be constant or they may depend on other variables. Since Hamilton (1989)'s seminal work, a large literature has developed applying regime switching models to financial time series variables where there is evidence of changing behaviour of the series across business cycles. Gray (1996), Bekaert, Hodrick and Marshall (2001) and Ang and Bekaert (2002b and 2002c) find strong evidence of regimes in US and international short-term interest rate data. Ang and Bekaert (2002a and 2002b), Ang and Chen (2002), Garcia and Perron (1996), Guidolin and Timmermann (2007), Perez-Quiros and Timmermann (2000), Turner et al. (1989) and Whitelaw (2000) all report evidence of regimes in stock or bond returns.

In the strategic asset allocation literature, Ang and Bekaert (2002a), Ang and Bekaert (2004) and Guidolin and Timmermann (2007) consider the asset allocation implications of regime switching models, because they capture many of the properties of asset returns that emerge from the empirical studies such as having regimes with very different mean, volatility and correlations across assets. Ang and Bekaert (2002a) use a two-state model to evaluate the claim that the home bias observed in holdings of international assets can be explained by return correlations that increase in bear markets. Guidolin and Timmermann (2007) use a four regime model that has a crash state capturing large negative returns and a bull state capturing large positive returns. They claim that a four regime model best captures the joint distribution of stock and bond returns. However they do not examine the performance of optimal asset allocation under different regime models. Regime switching models are also useful in capturing fat tails and skewness in the distribution of asset returns (Guidolin and Timmermann (2008)).

In light of these papers, we consider a two-state Markov switching mean-variance model close in spirit to Ang and Bekaert (2004). They conclude that the presence of regimes with different correlations and expected returns is exploitable within an international equity allocation framework. In addition to examining Ang and Bekaert’s (2004) cross-country asset allocation using an extended sample period, our paper contributes the strategic asset allocation literature by also investigating the regime-dependent cross-sector allocation, motivated by recent literature that discusses the increasing importance of sector diversification.

2.2 Cross-region, cross-sector allocation

The risk reduction benefits of the international diversification of equity portfolios have been accepted for a long time among academicians (e.g., Solnik (1974)). However knowing what factors drive the co-movement in stock returns across countries has long challenged both academics and professional portfolio managers. A number of studies (e.g., Grinold et al. (1989), Heston and Rouwenhorst (1994)) examine the relative importance of country and industry factors in explaining the cross section of expected returns and establish country factors as the major influence on equity returns. Heston and Rouwenhorst (1994) decompose stock return volatility into pure country and industry sources of variation and clearly document the dominance of country-specific effects. Griffin and Karolyi (1998) find that when emerging markets are included in the sample, the proportion of portfolio variance explained by the time-series variation in pure country effects is higher than previously documented, which again indicates investors would be better off in terms of risk reduction if they pursued a geographic diversification strategy rather than an industry one.

The relative importance of the country factor has been challenged, however, by Baca et al. (2000), Cavaglia et al. (2000), Cavaglia and Moroz (2002) and Chen et al. (2006). These studies conclude that the importance of sector factors has grown to exceed that of country factors in both developed and emerging markets in recent years. They attribute these empirical findings to an increased level of international capital markets integration, which blurs national borders and hence diminishes the significance of country effects. However, Brooks and Del Negro (2004) argue that the rise in industry effects is simply a temporary phenomenon associated with the information technology bubble, rather than a reflection of greater economic integration across countries.

On balance, the studies suggest that industry factors have become as important as, if not more than, country factors. Nevertheless, most of the studies discussed above do not consider the optimal asset allocation implications of the growing importance of sector effects. As the financial markets become increasingly integrated, it makes intuitive sense to ask whether sector asset allocation could add benefits to the widely practiced international asset allocation.

A relatively unexplored area of sector asset allocation is with regard to sector rotation strategies. Conventional market wisdom posits that different sectors perform differently in various stages of the business cycle. Most professional investors seem to agree that sector rotation strategies can be extremely profitable with good market timing skills (e.g., Stovall (1996)). However, academic research has not yet rigorously tested whether investors can profit from sector rotation strategies. The major obstacle has been to select appropriate indicators that identify the exact jumps or turning points of business cycles or financial conditions contemporaneously. Previous studies are mostly heuristic rather than scientific in determining these indicators. Sorensen and Burke (1986) use relative strength analysis for 43 industries and find that an industry momentum-based rotation strategy produces abnormal profits. Using macroeconomic variables such as the default premium, maturity premium, and aggregate dividend yield, Beller et al. (1998) create an industry trading strategy that earned economically significant profits. Sassetti and Tani (2006) find that the sector rotation strategy is profitable using a number of technical indicators including relative strength and moving averages. Conover et al. (2005, 2008) find that cyclical stocks prosper during expansive monetary policy periods, while a restrictive environment favours defensive stocks and a sector rotation strategy based on the monetary policy produces excess returns.

Most of these studies hold a common view that market timing is driven by exogenous variables. Therefore the success of sector rotation strategies solely relies on the appropriate choice of exogenous indicators. However, using exogenous indicators suffers reverse causation or omitted variables problems. For instance, exogenous indicators may be reversely determined by the return generation process, or both can be driven by some omitted variables. Hence such strategies could imperfectly capture the true underlying dynamics of time-varying sector performances. To address these potential problems, we propose a Markov switching process that endogenously derives the turning points or jumps of regime cycles from the statistical feature of stock returns, and therefore is robust to reverse causation or omitted variables problems. As discussed in section 2.2, there is mounting evidence that suggest the Markov switching process can capture the time-varying distributions of stock returns.

3 Data

In the cross-region asset allocation, we focus on a group of developed equity markets that constitute the MSCI world index for an US based institutional investor. These developed equity markets include North-America, UK, Japan, large European markets, small European markets and the Pacific ex-Japan. Table 1 details all countries involved. All data are sourced from the MSCI and the sample period is May 1975 through to March 2010.

In the cross-sector allocation, we focus on ten broad sectors classified by the MSCI Global Industry Classification Standard (GICS), namely Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Healthcare, Financials, Information Technology, Telecommunications and Utilities. All data come from the MSCI and the sample period is from January 1995 to March 2010.

Table 2 and 3 report some simple characteristics of the regional and sectoral returns data. We measure all returns in US dollars. These return properties are expected to play a large role in determining mean-variance based asset allocations. For example, it is immediately apparent that the use of historical data may lead to relatively small weights to Japan, because it has witnessed relatively low returns (underperform the world market by 0.06% per month) in the past few decades with relatively high volatility (6.31% monthly). Similarly, a simple mean-variance optimization may lead to some large weights to small European economies, as they have relatively higher returns (outperform the world market by 0.28% per month) with low return volatility (4.90% monthly). It is also interesting to note that the IT sector has the highest mean return during the sample period, although this sector has experienced a major boom and bust.

4 Regime-Switching Model

4.1 Description of the model

To maintain the parsimony of the model, we adopt Ang and Bekaert's (2004) approach, which assumes that there is only one world regime, which drives all other regions/sectors. Weakening this assumption by estimating specific regimes in each region/sector separately makes the number of parameters infeasible for estimation. The equation for the world equity return, in excess of the US T-bill rate is:

$$y_t^w = \mu^w(s_t) + \sigma^w(s_t)\varepsilon_t^w, \tag{1}$$

Here $\mu^w(s_t)$ denotes the conditional mean (expected returns) and $\sigma^w(s_t)$ denotes the conditional variance (volatility). Where ε_t^w denotes the distribution of the random variable, which is assumed to be normal, the same in all regimes. We assume the world excess equity returns have two unobserved regimes, regime i and j. This number of states may be restrictive, but including more regimes poses extreme computational problems. Two states should capture the main effects of higher order moments in equity returns. Therefore $\mu^w(s_t)$ and $\sigma^w(s_t)$ can take different values depending on the realization of the regime variable s_t.

The regimes s_t follow a two-state Markov chain with transitional matrix:

$$\begin{pmatrix} P & 1-P \\ 1-Q & Q \end{pmatrix}$$

which can be characterized by two transitional probabilities:

$$P = p(\, s_t = i \mid s_{t-1} = i)$$

$$Q = p(\, s_t = j \mid s_{t-1} = j). \tag{2}$$

If investors know the regime, the expected excess return and volatility for the world market in the next period will be either:

$$e_i^w = P\mu^w(s_{t+1} = i) + (1-P)\mu^w(s_{t+1} = j), \tag{3}$$

$$\Sigma_i^w = P\big(\sigma^w(s_{t+1} = i)\big)^2 + (1-P)\big(\sigma^w(s_{t+1} = j)\big)^2$$

$$+ P\,(1-P)[\mu^w(s_{t+1} = j) - \mu^w(s_{t+1} = i)]^2, \tag{4}$$

when the regime realization today is $s_t = i$, or

$$e_j^w = (1 - Q)\mu^w(s_{t+1} = i) + Q\mu^w(s_{t+1} = j) \tag{5}$$

$$\Sigma_j^w = (1 - Q)\left(\sigma^w(s_{t+1} = i)\right)^2 + Q\left(\sigma^w(s_{t+1} = j)\right)^2$$
$$+ Q\,(1 - Q)[\mu^w(s_{t+1} = j) - \mu^w(s_{t+1} = i)]^2, \tag{6}$$

when the regime realization today is $s_t = j$.

The first component in equations (4) and (6) is a weighted average of the conditional variances in the two regimes; the second component is a jump component that arises because the conditional mean is different across regimes.

In contrast to Ang and Bekaert (2004), which does not allow regime switching in idiosyncratic region/sector behaviour, we model the individual region/sector excess returns y_t^v, using a regime switching CAPM model:

$$y_t^v = \mu^v(s_t) + \beta^v(s_t)\,\mu^w(s_t) + \beta^v(s_t)\,\sigma^w(s_t)\varepsilon_{t+1}^w + \bar{\sigma}^v(s_t)\varepsilon_{t+1}^v \tag{7}$$

Where $\beta^v(s_t)$ denotes different correlations between the individual region/sector excess returns and the world excess equity returns in different regimes, $\mu^v(s_t)$ denotes alphas in regime one and regime two, and $\bar{\sigma}^v\ (s_t)$ denotes the regime-dependent region/sector's idiosyncratic volatility. With regime switches, this model captures time-variation in expected returns, volatilities and correlations, all driven by the world regime variable.

Ang and Bekaert (2004) state that accommodating regime switching alphas, regime switching betas, and/or regime switching idiosyncratic volatilities fit the data better, yet some estimations do not behave well in their model, and it is often difficult to make inferences about the regime in such models. To overcome this problem, we do not allow regional and sector returns to affect the regime realization process and hence estimate equation (7) after regime probabilities are inferred from the world return process. Regime probabilities play a critical role in the estimation of regime switching models, which uses maximum likelihood techniques.

We follow Hamilton (1989)'s maximum likelihood estimation algorithms to estimate the parameters in equation (1) to (6) and regime probability, which is the probability that tomorrow's regime is a particular regime given current and past information τ_t, that is, the world equity returns data $\breve{y}_T = (y_t^w y_{t-1}^w \dots y_1^w y_0^w)$. Because the regimes are unobservable, their effects and incidence must be inferred from the data. Maximizing the likelihood function that accounts for all possible regime sequences is the most direct manner to accomplish this task. Let the parameters of the likelihood to be θ, the likelihood function is:

$$f(\check{y}_T;\ \theta) = \prod_{t=1}^{T}\left(\sum_{i=1}^{2} f(y_t\ |\tau_{t-1}, S_t = i; \theta) p(S_t = i\ |\tau_{t-1}; \theta)\right) \tag{8}$$

4.2 Model Estimation

To reliably estimate the model parameters, we need to use a reasonably long period of time. We estimate the regime switching region model with the sample period from May 1975 to December 2000. We omit the subsequent sample period from January 2001 to March 2010 for out-of-sample tests. Table 4 contains the estimation results for the region model in equations (1)-(7). The first regime is a bull market, where world excess returns are expected to yield 0.95% per month (11.36% annualized), with 2.92% (10.13% annualized) volatility. On the other hand, the second regime, a bear market, has a much lower mean world excess return (0.09% per month, 1.06% annualized) and higher volatility (5.16% per month, 17.86% annualized). The transitional probabilities P and Q are 0.90 and 0.86 respectively, which indicate that, over the sample period, the probability of going into a bull (bear) market next month is 90% (86%), conditional on the current regime being a bull (bear) market.

The expected durations of these two regimes are ten and seven months respectively.[1] Regional betas are estimated with a high level of statistical significance in both regimes, and their magnitudes seem economically appealing. Betas for each region are quantitatively similar in both regimes, apart from the Pacific ex-Japan region which is less correlated (beta = 0.89) with the world market during the normal period but takes on a higher systematic risk (beta = 1.04) in bear markets. With the rather low average returns during the sample period, surprisingly, Japan has the highest betas in both regimes. Nevertheless, the large negative alpha (-0.55% per month) would result a much lower expected return for Japan during bear markets. The alpha parameters are not significantly different from zero for all regions, except the small European economies (0.41% per month, t=2.14). Note that alphas for North America and United Kingdom are actually positively larger (0.22% and 0.40% per month) in bear markets than those in bull markets (0.16% and 0.32% per month), making equities in these regions more attractive in the bad times. Similar to the world return process, idiosyncratic volatilities for each region are lower in regime one and higher in regime two. Japan and Pacific ex-Japan exhibit the highest idiosyncratic volatilities in both regimes

[1] The expected duration of a regime can be calculated as 1/ (1-P), where P is the transitional probability.

whereas the lowest idiosyncratic volatilities are found in North America, followed by small European economies.

The estimation procedure also yields the regime probabilities, which infers the prevailing regime in each month. We need to estimate the regime probabilities for both in-sample (1975 to 2000) and out-of-sample period (2001 to 2010) for performance evaluation purposes. Panel A in Figure 1 shows the full-sample accumulated buy and hold returns of \$1 from May 1975 to March 2010. Panel B shows the *ex ante* (filtered) and *ex post* (smoothed) regime probabilities. The *ex ante* probability $p(s_t = 1|I_{t-1})$ is the probability that the regime next month is the bull market regime given past and current information I up to time t, the smoothed probability $p(s_t = 1|I_T)$ is the probability that the regime next month is the bull market regime given all of the information I present in the sample period T, that is, from May 1975 to March 2010. The dotted line represents several economic contraction periods during the sample period identified by the National Bureau of Economic Research (NBER)[2]. Some of the financial bear markets identified by the regime-switching algorithms coincide with these economic contraction periods, namely the 1980s and 1990s Savings and Loans Crisis, the 2000 Dot-Com Bubble and the recent Global Financial Crisis. The model also identifies other crises restricted to the financial market such as the 1987 crash.

To estimate the parameters for the sector model, we use ten years of data (from January 1995 to December 2004). The remaining five years (from January 2005 to March 2010) are left out for an out-of-sample test. Table 5 contains the estimation results for the sector model in equations (1)-(7). During this period, the world excess returns are expected to generate 0.98% per month (11.77% annualized), with 2.26% (7.84% annualized) volatility. Whereas the bear market shows a negative expected return of -0.14% per month (-1.72% annualized) and a higher monthly volatility of 4.92% (17.06% annualized). The negative expected return in the bear market regime may seem extreme and appear to be incompatible with equilibrium arguments by which risky assets should earn a positive risk premium. However, because we have a relatively short sample period, it is not unreasonable to expect equity returns to be lower than the risk-free return. Compared to the bull regime, all sectors are less correlated with the world market in the bear regime except IT (beta=1.89) and Telecommunications (beta=1.15), primarily because the only bear regime in the estimation period is the Dot-Com Bubble, where the influence of IT and Telecommunications sectors on the world return

[2] Business cycle dates identified by NBER can be found at http://www.nber.org/cycles/cyclesmain.html.

process increased dramatically. Consumer Staples, Healthcare and Utilities, commonly recognized as defensive sectors, have rather low correlations with the world market during bad times with betas equal to 0.42, 0.48 and 0.40 respectively. Alphas are lower in the bear regime for most of the sectors. It is surprising to see that IT gets loaded with a high alpha of 0.75% per month (not statistically significant) in the bear regime. However, IT has the highest volatility of all the sector returns in both regimes, which the model fits through a high beta and a high idiosyncratic volatility (the highest idiosyncratic volatility of all sectors). As expected, idiosyncratic volatilities are higher in the bear regime for all sectors.

5 Asset Allocation and Performance Results

5.1 Asset Allocation

To derive the expected returns and covariance matrix for the returns on regions/sectors, we let the vector of excess returns of six regions and ten sectors, conditional on today's regime, be denoted by $e_i = e(s_t = i)$ (with i denoting the current regime), and let variance covariance matrices be denoted by $\Sigma_i = \Sigma(s_t = i)$.

Since the mean of the world excess return switches between regimes, the expected excess return of region/sector v is given by $\mu_i^v + \beta_i^v\, e_i^w$ for the current regime i, where e_i^w are given in equation (3) and (5). Let

$$\mu_i = \begin{pmatrix} \mu_i^1 \\ \vdots \\ \vdots \\ \mu_i^N \end{pmatrix}, \quad \beta_i = \begin{pmatrix} \beta_i^1 \\ \vdots \\ \vdots \\ \beta_i^N \end{pmatrix}$$

for N regions/sectors. Hence the expected return vector is given by:

$$e_i = \mu_i + \beta_i e_i^w \tag{9}$$

Therefore, expected returns differ across individual regions/sectors through their different betas and alphas with respect to the world market.

The variance covariance matrix has three components. First, there is an idiosyncratic part that we capture in a matrix $\mathrm{v_i}$ for the current regime i, where $\mathrm{v_i}$ is a matrix of zeros with $(\bar{\sigma}_i^v)^2$ along the diagonal. Second, the differences in systematic risk across the different regions/sectors and the correlations are driven by the variance of the world market and the betas as in any factor model. Because the world market variance and the betas next period depend on the realization of the regime, we have two possible variance matrices for the unexpected returns next period:

$$\Omega_i = (\beta_i \beta_i')(\sigma_i^w)^2 + v_i \tag{10}$$

Third, the actual covariance matrix today takes into account the regime structure, in that it depends on the realization of the current regime and it adds a jump component to the conditional variance matrix, which arises because the conditional means change from one regime to the other. As a consequence, the conditional covariance matrices can be written as:

$$\Sigma_1 = P\Omega_1 + (1-P)\Omega_2 + P(1-P)(e_1 - e_2)(e_1 - e_2)'$$
$$\Sigma_2 = (1-Q)\Omega_1 + Q\Omega_2 + Q(1-Q)(e_1 - e_2)(e_1 - e_2)' \tag{11}$$

where the subscripts indicate the current regime.

To implement mean-variance optimization, we also need to make assumption about the risk-free rate. We assume the risk-free rate to be constant and the arithmetic average of 1 month T-bill rate across the estimation period (1975~2000). We do not allow the risk-free rate to change over-time because short term shocks to the T-bill rate may introduce measurement errors to the risk-free rate estimation.

One obvious extension discussed in Ang and Bekaert (2004) is to impose constraints on asset allocations. This is because firstly mean-variance portfolios, based on historical data, may be quite unbalanced (see Black and Litterman (1992)), and secondly, in practice institutional investors have an intelligence overlay rather than just applying weights based on the straight outputs. We choose to impose a short sell constraint, and a benchmark constraint that keeps asset allocations close to their market capitalization weights in the MSCI world index. The short constraint requires weights to be positive, whereas the benchmark constraint does not allow the optimal region weights to deviate from their MSCI average weights by more than 10%.

Panel A and B of Table 6 show the regime-dependent expected excess returns, covariances and correlations for the regional allocation. The expected excess returns are lower for all regions in the bear regime, with Japan in particular generating a negative expected excess return (-3.59% per annum). In line with the results of Longin and Solnik (2001) and Ang and Bekaert (2002a), our results also indicate that international equity returns are more highly correlated with each other in bear markets than in bull markets. For instance, most of cross country correlations are less than 0.5 in the bull markets and almost all correlations become greater than 0.5 in the bear markets. The average correlation has increased from 0.43 to 0.59.

Panel C of Table 6 shows the tangency portfolios weights in both regimes and static mean-variance optimization at an interest rate of 6.67%, the sample average. When no constraints are imposed, in the bull regime, the portfolio under-weights North America, UK, Japan, Pacific ex-Japan relative to their average MSCI world index weights over the sample period. The high volatility of UK, Japan and Pacific ex-Japan markets is the main problem. Our model also slightly over-estimates the correlation between North America and Europe small returns relative to the data, which may explain the underweight in North America. Both large and small European economies are over-weighted. In the bear regime, investors switch towards the markets with less volatility and higher expected returns, namely, North America, UK and small European countries. Although North America gets assigned a weight of 74%, it does not mean the portfolio is now home biased because the investors also invest heavily in UK (39%) and small European countries (53%).

The ability to invest heavily in the less volatile markets comes from a large short position on Japan, almost 70%. However, it is probably unrealistic to implement this short position in practice. The optimal portfolio under the static mean-variance analysis sits in between the two regime-dependent portfolios, but under-weights almost all regions relative to their MSCI world index weights except small European countries. The optimal portfolios weights in both regimes and static analysis do not change qualitatively under the short constraint and the benchmark constraint. All portfolios still underweight Japan and overweight small European countries. Note that under the benchmark constraint, regime-dependent and static portfolios have exactly the same allocation on Japan and small European countries (12% and 17% respectively). The benefits of the regime switching allocation therefore only come from investors tilting towards North America and UK and moving away from large European countries and Pacific ex-Japan during the bear markets.

Panel A and B of Table 7 show the regime-dependent expected excess returns, covariances and correlations for the sector allocation. Energy, Healthcare and IT show the highest expected excess returns in both regimes, whereas Materials, Industrials, Telecommunications and Utilities are clearly the losers. It is unreasonable to expect IT to generate the highest returns in the bear regime (6.23% per annum), particularly when the sample period covers the

Dot-Com Bubble. The main reason is probably the use of simple returns in our analysis.[3] However, we do not use logarithmic returns because, as stated in Ang and Bekaert (2004), they complicate the link between the regime switching model and expected returns used in the asset allocation optimization and simple returns provide a tighter link with a standard CAPM model. Unlike correlations between regions, sector correlations do not increase universally in bad times. For instance, the correlation between Consumer Staples and Telecommunications fall from 0.56 in regime one to 0.21 in regime two. Out of 45 correlation pairs, 12 pairs demonstrate decreased correlations in the bear markets, mainly concentrated within Consumer Staples, IT, Healthcare, and Telecommunications sectors. The average cross sector correlation has only increased from 0.48 to 0.52. This feature of sector allocation provides good reason on why diversification cross sectors is important not only in good times but also in bad times.

Panel C of Table 7 shows the tangency portfolios weights in both regimes and static mean-variance optimization at an interest rate of 6.67%. With no constraints enforced, the portfolio in bull markets over-weights Energy, Industrials, Consumer Staples, Healthcare and Utilities relative to their average MSCI world index weights over the sample period, but under-weights Materials, Consumer Discretionary, Financials, IT and Telecommunications. The high volatility is still the main problem, although IT yields the highest expect returns. In regime two, the portfolio borrows money from shorting Materials (-40%), Industrials (-197%) and telecommunications (-49%) massively and invests heavily on few sectors including Energy (93%), Consumer Discretionary (56%), Consumer Staples (58%), Healthcare (104%) and IT (66%). The large short position on Industrials primarily comes from its high correlation with Consumer Discretionary and IT sectors (0.88 and 0.73). This large short position is perhaps neither feasible nor sensible to implement in a practical asset allocation program. This issue can be addressed when constraints are imposed. The static mean-variance optimal portfolio also favours Energy, Consumer Staples, Healthcare and IT and is highly averse to Industrials and Telecommunications. When short-sell is not allowed, both regime-dependent and static optimal portfolios only invest in Energy, Consumer Staples, Healthcare and IT. The benchmark constraint is so restrictive that the regime-dependent portfolios only deviate from the static optimal portfolio marginally. More aggressive investors can obviously modify the constraint to be less restrictive but we will not demonstrate other restrictions in our simulation.

[3] Simple returns tend to introduce an upward bias.

5.2 Performance of Regime Switching Asset Allocation

To evaluate whether regime switching asset allocation adds value to standard mean-variance optimization, we simulate a time-series of returns generated from a regime switching strategy that switches between holding two regime-dependent optimal portfolios and compare it to the returns generated from holding a static mean-variance efficient portfolio. We estimate the returns of these two strategies both in-sample and out-of-sample, with \$1 to start. In-sample simulation assumes the investor knows the parameters estimated from the estimation period (May 1975 ~ Dec 2000 for region model, Jan 1995 ~ Dec 2004 for sector model) and starts trading in May 1975 for the region model and Jan 1995 for the sector model. In the out-of-sample analysis, the model is estimated up to the end of the estimation period and the regime-dependent and static mean-variance weights are computed using information available only from the estimation period. The model is not re-estimated during the out-of-sample period (Jan 2001 ~ Mar 2010 for region model and Jan 2005 ~ Mar 2010 for sector model). The regime strategy also requires the realization of regime in each month, which can be inferred from the *ex-ante* regime probability $p(s_t = 1|I_{t-1})$. When this probability is larger than 0.5, the investor classifies the regime as 1, otherwise the regime is 2. The performance criterion is the ex-post Sharpe ratio realized by the various strategies.

Panel A of Table 8 shows the in-sample average returns, standard deviations and Sharpe ratios realized by the static mean-variance, regime-dependent strategies and the MSCI world index for asset allocation across regions, when various constraints are considered. Compared to the world market portfolio and the static strategy, the regime-dependent strategy has higher return volatilities, compensated by higher average returns under various constraints. The regime switching strategy has the highest *ex-post* Sharpe ratio not only when no constraints are imposed but also when the short constraint and the benchmark constraint are imposed, although the out-performance is inevitably lower. The regime-dependent strategy does so well because over this sample period the US and small European markets generate very large returns and Japan performs very poorly. The reason for the under-performance of the world market portfolio is the presence of a relatively large Japanese equity allocation in the world market. It is perhaps more important to examine the out-of-sample performance because no hindsight bias is introduced into the simulated returns of both strategies.

Panel B shows, over the out-of-sample, with no constraints, the regime-dependent strategy's average return is 9.14%, double the average return of the static portfolio (4.55%). It is almost triple the return of the world market portfolio (3.54%). The regime-switching portfolio's average return Sharpe ratio is 0.31, more than double the static optimal portfolio Sharpe ratio (0.13), where the world market portfolio only produces a Sharpe ratio of 0.07. The extremely low Sharpe ratios for all portfolios are results of the Dot-Com Bubble and the Global Financial Crisis in the sample period. Note that under the short constraint and the benchmark constraint, the regime-dependent strategy is so successful that it delivers the lowest volatility and the highest returns in the out-of-sample period, compared to the world market portfolio and the static portfolio.

Figure 2 and 3 show how wealth cumulates over time in these strategies with no constraints and the benchmark constraint. Panel A shows the out-performance of the regime-dependent strategy is particularly striking in the last five years of the in-sample period, where as Panel B shows that the regime switching portfolio performs better during the market boom but poorly during the bear market, particularly the recent Global Financial Crisis. This is primarily because the regime-dependent portfolio invests heavily in UK in bad times, which has performed particularly well during the bear regime in the parameter estimation period but is the under-performer in this financial crisis.

The performance of the regime dependent portfolio and static optimal portfolio for the sector allocation is shown in Table 9. In the in-sample analysis, the static optimal portfolio realize slightly lower returns, but do so at substantially lower risk than the regime dependent sector allocations. Hence the regime dependent portfolio leads to lower in-sample Sharpe ratio than the static optimal portfolio. The under-performance is expected because the regime probabilities might be estimated imprecisely in the earlier trading period due to the limited data availability for estimation. With more data becoming available, the regime switching model is able to estimate the regime probability with higher precision.

Panel B shows the portfolio performance over the out-of-sample period, from January 2005 to March 2010. With no constraints imposed, the regime dependent strategy delivers an impressive14.47% average annual return (13.49% annualized volatility) in a relatively bearish period, where the static portfolio and the world market portfolio only generate an average return of 6.93% and 1.59% (11.17% and 15.54% annualized volatility) respectively. The regime switching portfolio's Sharpe ratio is 0.88, more than double the Sharpe ratio of the static strategy, whereas the world market portfolio even produces a slightly negative Sharpe ratio (-0.07) during this traumatic period. The regime dependent portfolio consistently out-performs the world market and the static optimal portfolio when the short constraint and the benchmark constraint are imposed. The extremely successful performance of the regime dependent strategy comes from the ability of the model to correctly identify the defensive sectors, such as Energy, Consumer Staples and Healthcare, which hedge against high volatilities and low returns so well in the bear regime.

Figure 4 and 5 present results for the wealth accumulated over the in-sample and out-sample period, under zero constraints and for the benchmark constraint scenario. Panel A shows that the regime dependent strategy yields lower returns than the static portfolio during the first three years of the in-sample period, probably due to an imprecise classification of regimes. In Panel B, it is interesting to note that, particularly over the second half of the out-of-sample period, wealth accumulated from the regime dependent strategy is almost unaffected while the world market is deeply in turmoil. Unlike the regional allocation, even the static mean-variance sector allocation produces a defensive portfolio that is more or less decoupled from the world market portfolio in the bear regime.

It is worthwhile to compare the performance of static and regime switching asset allocation across regions and across sectors. Table 10 shows the average returns, standard deviations, betas and Sharpe ratios generated by the static and the regime dependent strategies for region and sector allocations for each year from 1995 to 2009. On average, cross-sector allocation provides higher returns, lower risks, lower betas (correlations with the world market) and higher Sharpe ratios than cross-region allocation. In the cross-region allocation program, the regime-dependent portfolio produces a higher Sharpe ratio (0.63), together with a higher beta (1.12) than the static strategy. The regime switching strategy out-performs the static strategy in Sharpe ratio in ten out of the total 15 years, including the bear market from 2000 to 2002. In the cross-sector allocation, the regime-dependent strategy produces a higher Sharpe ratio (0.75), however with a lower beta (0.36) than the static strategy. The regime switching strategy in sector allocation also out-performs the static strategy in ten out of 15 years. Figure 6 illustrates that, from January 1995 to March 2010, the regime dependent sector allocation produces the highest return compared to other allocation programs and provides a defensive portfolio that hedges against low returns and high volatilities in the bear markets. Most remarkably, the regime switching sector allocation yields an average return of only -0.08% per month and a monthly volatility of 6.72%, compared to the world market average monthly return of -3.98% and volatility of 6.89% in year 2008, the largest stock market decline in a single year since the Great Depression.

6 Conclusion

Static mean-variance asset allocation fails to exploit the characteristics of high volatility and low equity returns in bad times. Although in high-volatility environments regional returns tend to be more correlated collectively, this is not the case for the sector returns. Thus diversification across sectors should provide additional benefits. This salient feature of regional and sector returns should not be ignored by active portfolio managers.

We demonstrate how the regime switching model could match this feature and improve the performance of asset allocation across regions and sectors. However, the out-performance could be sample specific and has not taken into account of market frictions such as transaction costs and taxes. Nevertheless, the regime switching strategy should be robust to transaction costs because the probability of staying within the same regime is relatively high and portfolio turnover is low.

Two robust conclusions can be drawn from our current results. First, both regional and sector regime dependent portfolios have the potential to out-perform the regional and sector static mean-variance portfolios. Particularly, the regime dependent sector allocation delivers the best performance, because the portfolio heavily invests in defensive sectors that have low correlations and hence provide considerable diversification benefits in bear markets. Second, cross-sector optimal allocation provides higher returns, lower risks, lower correlations with the world market and higher Sharpe ratios than cross-region optimal allocation.

One limitation of this research is that the sector allocation model uses a relatively short and perhaps unique sample period. One obvious extension would be to examine the regime switching sector allocation using a longer sample period and to re-estimate the model more frequently.

The performance of the regime switching strategy could possibly be improved by incorporating the following extensions. First, we only consider a two regime structure in our study. Further research can look at the implications on the performance of cross-region and cross-sector allocation using three or four regime structures, which might fit equity returns data better (Guidolin and Timmermann (2007)).

Second, because we only use monthly returns data, it would be interesting to examine regime switching behaviour on a weekly basis, which might signal more timely information, especially during the start of the bear markets when immediate diversification is needed the most, although using weekly data might create more frequent switches between regimes and generate much higher portfolio turnover and trading costs.

Finally, another viable alternative is to infer the regimes from macroeconomic (or consensus forecasts) variables such as the inflation rate (or the expected inflation rate) that can influence stock returns (Fama and Schwert (1977), Nelson (1977) and Jaffe and Mandelker (1976)). For example, a period of high inflation may prelude low stock market returns. Therefore, a regime switching strategy that trades based on macroeconomic regimes might offer additional benefits.

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Table 1: Composition of Regional Returns

North America	UK	Japan	Europe Large	Europe Small	Asia Pacific ex-Japan
Canada			Italy	Austria	Australia
US			France	Belgium	Hong Kong
			Germany	Denmark	Singapore
				Finland	New Zealand
				Ireland	
				Netherlands	
				Norway	
				Spain	
				Sweden	
				Switzerland	

The table lists the country composition of the geographic returns. Within each geographic region, we construct monthly returns, value-weighted in US dollars.

Table 2: Descriptive Statistics - Regional Returns 1975 ~ 2010

Sample Moments	World	North America	UK	Japan	Europe Large	Europe Small	Pacific
Mean	0.47	0.53	0.64	0.41	0.62	0.75	0.70
Stdev	4.25	4.40	5.66	6.31	5.69	4.90	6.43
Skewness	-0.65	-0.62	-0.12	0.29	-0.42	-0.63	-1.04
Kurtosis	1.84	2.21	1.54	0.73	1.42	1.80	6.27
Beta		0.92	1.08	0.99	0.99	0.95	1.10

Correlation Matrix	World	North America	UK	Japan	Europe Large	Europe Small	Pacific
North America	0.88						
UK	0.72	0.58					
Japan	0.67	0.36	0.40				
Europe Large	0.76	0.58	0.60	0.47			
Europe Small	0.84	0.68	0.67	0.53	0.87		
Pacific	0.68	0.60	0.57	0.42	0.52	0.63	

The table reports summary statistics for the regional returns. Regional returns and standard deviation are expressed as simple returns at a monthly frequency in percentages. Regional returns are denominated in US dollars and are from MSCI and are in excess of the US 1-month T-bill return. The row labelled beta is the full-sample beta for each region's excess return with the world market excess return. The sample period for the regional returns is May 1975 to Mar 2010.

Table 3: Descriptive Statistics - Sector Returns 1995 ~ 2010

Sample Moments	World	Energy	Materials	Industrials	Consumer Discr	Consumer Staples	Health care	Financials	IT	Telecom	Utilities
Mean	0.16	0.59	0.26	0.14	0.12	0.31	0.41	0.05	0.62	-0.04	0.09
Stdev	4.27	5.08	5.48	4.78	5.02	3.35	3.81	5.71	7.96	5.42	3.42
Skewness	-0.89	-0.18	-0.66	-0.89	-0.39	-0.70	-0.49	-0.62	-0.30	-0.06	-0.95
Kurtosis	1.40	0.86	2.63	2.76	1.51	1.41	0.32	2.77	0.75	2.28	1.25
Beta		0.74	1.02	1.04	1.09	0.50	0.55	1.19	1.57	0.93	0.51

Correlation Matrix	World	Energy	Materials	Industrials	Consumer Discr	Consumer Staples	Health care	Financials	IT	Telecom	Utilities
Energy	0.62										
Materials	0.80	0.70									
Industrials	0.93	0.61	0.85								
Consumer Discr	0.93	0.48	0.75	0.89							
Consumer Staples	0.63	0.41	0.52	0.61	0.53						
Healthcare	0.61	0.35	0.39	0.52	0.47	0.69					
Financials	0.89	0.50	0.71	0.86	0.82	0.68	0.60				
IT	0.84	0.41	0.56	0.72	0.80	0.29	0.38	0.60			
Telecom	0.73	0.32	0.43	0.58	0.66	0.35	0.41	0.53	0.72		
Utilities	0.64	0.57	0.53	0.62	0.51	0.62	0.55	0.61	0.34	0.42	

The table reports summary statistics for the sector returns. Sector returns and standard deviation are expressed as simple returns at a monthly frequency in percentages. Sector returns are denominated in US dollars and are from MSCI and are in excess of the US 1-month T-bill return. The row labelled beta is the full-sample beta for each sector's excess return with the world market excess return. The sample period for the sector returns is Jan 1995 to Mar 2010.

Table 4 Regime Switching Region Model Parameter Estimates 1975 ~ 2000

A. Transitional probabilities

Measure	P	Q
Estimate	0.90	0.86
Std error	0.08	0.09

B. World Market

Measure	μ_1	μ_2	σ_1	σ_2
Estimate	0.95	0.09	2.92	5.16
Std error	0.32	0.58	0.44	0.70

C. Individual regions - Regime 1

Region Betas	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Estimate	0.90	0.99	1.16	0.88	0.88	0.89
Std error	0.06	0.12	0.12	0.10	0.07	0.12

Region Alphas	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Estimate	0.14	0.11	0.14	0.31	0.41	0.10
Std error	0.16	0.32	0.33	0.28	0.19	0.34

Idiosyncratic Volatilities	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Estimate	2.11	4.19	4.61	3.58	2.53	4.52
Std error	0.15	0.30	0.31	0.26	0.18	0.32

D. Individual regions - Regime 2

Region Betas	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Estimate	0.87	1.01	1.20	0.90	0.90	1.04
Std error	0.05	0.07	0.08	0.07	0.05	0.11

Region Alphas	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Estimate	0.22	0.40	-0.55	0.02	0.18	0.03
Std error	0.29	0.44	0.50	0.42	0.28	0.63

Idiosyncratic Volatilities	N Amer	UK	Japan	Eur lg	Eur sm	Pac
Estimate	2.82	4.46	4.70	4.28	2.86	6.07
Std error	0.28	0.43	0.49	0.41	0.28	0.62

All parameters are monthly and are expressed in percentages, except for the transition probabilities P and Q.

Table 5 Regime Switching Sector Model Parameter Estimates 1995~2004

A. Transitional probabilities

Measure	P	Q
Estimate	0.97	0.98
Std error	0.06	0.07

B. World Market

Measure	μ_1	μ_2	σ_1	σ_2
Estimate	0.98	-0.14	2.26	4.92
Std error	0.35	0.55	0.32	0.62

C. Individual sectors - Regime 1

Sector Betas	Energy	Materials	Industrials	Con. Discr	Con. Staples	Healthcare	Financials	IT	Telecom	Utilities
Estimate	0.65	1.19	1.09	1.15	0.67	0.76	1.16	1.35	0.72	0.54
Std error	0.19	0.15	0.08	0.08	0.10	0.15	0.09	0.24	0.12	0.11
Sector Alphas	Energy	Materials	Industrials	Con. Discr	Con. Staples	Healthcare	Financials	IT	Telecom	Utilities
Estimate	0.76	-0.28	-0.10	-0.18	0.40	0.61	-0.15	0.21	-0.10	0.13
Std error	0.46	0.36	0.20	0.21	0.25	0.38	0.22	0.58	0.31	0.27
Idiosyncratic Volatilities	Energy	Materials	Industrials	Con. Discr	Con. Staples	Healthcare	Financials	IT	Telecom	Utilities
Estimate	2.78	2.19	1.20	1.25	1.55	2.30	1.35	3.56	1.86	1.67
Std error	0.41	0.32	0.18	0.18	0.23	0.34	0.20	0.52	0.27	0.25

D. Individual sectors - Regime 2

Sector Betas	Energy	Materials	Industrials	Con. Discr	Con. Staples	Healthcare	Financials	IT	Telecom	Utilities
Estimate	0.62	0.84	0.94	1.08	0.42	0.48	1.07	1.89	1.15	0.40
Std error	0.10	0.09	0.05	0.05	0.08	0.09	0.07	0.13	0.12	0.08
Sector Alphas	Energy	Materials	Industrials	Con. Discr	Con. Staples	Healthcare	Financials	IT	Telecom	Utilities
Estimate	0.23	-0.31	-0.18	-0.01	-0.02	0.32	0.17	0.75	-0.04	-0.19
Std error	0.50	0.44	0.23	0.24	0.42	0.44	0.35	0.63	0.57	0.38
Idiosyncratic Volatilities	Energy	Materials	Industrials	Con. Discr	Con. Staples	Healthcare	Financials	IT	Telecom	Utilities
Estimate	4.23	3.74	1.97	2.05	3.55	3.77	2.94	5.38	4.85	3.25
Std error	0.49	0.43	0.23	0.24	0.41	0.44	0.34	0.63	0.56	0.38

All parameters are monthly and are expressed in percentages, except for the transition probabilities P and Q.

Table 6 Regime-Dependent Region Model Asset Allocation						
	N Amer	UK	Japan	Eur lg	Eur sm	Pac
A. Regime-dependent excess returns						
Regime 1	11.06	11.61	13.62	12.86	14.07	10.44
Regime 2	4.86	7.38	-3.59	2.52	4.43	3.00
B. Regime-dependent covariances and correlations						
Regime 1						
N Amer	1.57	***0.48***	***0.50***	***0.49***	***0.59***	***0.43***
UK	1.11	3.37	***0.38***	***0.37***	***0.45***	***0.33***
Japan	1.30	1.45	4.27	***0.39***	***0.47***	***0.34***
Eur lg	0.99	1.10	1.30	2.59	***0.46***	***0.33***
Eur sm	0.99	1.10	1.30	0.98	1.77	***0.40***
Pac	1.04	1.16	1.36	1.03	1.03	3.74
Regime 2						
N Amer	3.08	***0.63***	***0.66***	***0.61***	***0.71***	***0.55***
UK	2.54	5.30	***0.58***	***0.54***	***0.63***	***0.49***
Japan	3.02	3.50	6.80	***0.57***	***0.66***	***0.51***
Eur lg	2.27	2.64	3.14	4.47	***0.61***	***0.48***
Eur sm	2.27	2.63	3.13	2.36	3.30	***0.55***
Pac	2.61	3.03	3.60	2.71	2.70	7.25
C. Tangency portfolio weights						
MSCI Average	0.50	0.09	0.22	0.08	0.07	0.06
C1. No constraints						
Regime 1	0.28	0.04	0.01	0.18	0.46	0.03
Regime 2	0.74	0.39	-0.69	0.04	0.53	-0.02
Static	0.48	0.05	0.03	-0.06	0.60	-0.10
C2. Short constraint						
Regime 1	0.28	0.04	0.01	0.18	0.46	0.03
Regime 2	0.47	0.28	0.00	0.00	0.25	0.00
Static	0.45	0.02	0.02	0.00	0.51	0.00
C3. Benchmark Constraint						
Regime 1	0.41	0.07	0.12	0.18	0.17	0.06
Regime 2	0.60	0.19	0.12	-0.03	0.17	-0.04
Static	0.56	0.08	0.12	0.12	0.17	-0.04

We report the regime-dependent means and covariances of excess returns implied by the estimates of the Regime-Switching Region Model in Table 4. Panel A and B report the regime-dependent excess return means and covariances, where we list correlations in the upper-right triangular matrix. All numbers are listed in percentages, and are annualized. Panel C reports the mean variance efficient (MVE) (tangency) portfolios, computed using an interest rate of 6.67%, which is the average 1-month T-bill rate over the sample. The MSCI Average denotes the average MSCI world index weight of each region across the sample. The short constraint requires weights to be positive, whereas the benchmark constraint does not allow the optimal region weights to deviate from their MSCI average weights by more than 10%.

Table 7 Regime-Dependent Sector Model Asset Allocation

	Energy	Materials	Industrials	Con. Discr	Con. Staples	Healthcare	Financials	IT	Telecom	Utilities
A. Regime-dependent excess returns										
Regime 1	16.58	10.19	11.07	10.89	12.34	15.88	11.39	17.79	6.96	7.70
Regime 2	1.83	-5.00	-3.51	-1.67	-0.86	3.09	0.53	6.23	-2.13	-2.85
B. Regime-dependent covariances and correlations										
Regime 1										
Energy	1.19	***0.45***	***0.46***	***0.41***	***0.22***	***0.22***	***0.26***	***0.20***	***0.38***	***0.44***
Materials	0.59	1.45	***0.86***	***0.79***	***0.36***	***0.16***	***0.69***	***0.42***	***0.36***	***0.31***
Industrials	0.48	0.98	0.90	***0.88***	***0.48***	***0.33***	***0.77***	***0.54***	***0.52***	***0.44***
Consumer Discr	0.45	0.95	0.84	1.00	***0.60***	***0.39***	***0.75***	***0.54***	***0.52***	***0.44***
Consumer Staples	0.18	0.32	0.35	0.45	0.56	***0.64***	***0.59***	***0.40***	***0.56***	***0.51***
Healthcare	0.24	0.20	0.31	0.39	0.48	0.99	***0.52***	***0.37***	***0.54***	***0.36***
Financials	0.28	0.85	0.75	0.77	0.45	0.53	1.05	***0.41***	***0.52***	***0.57***
IT	0.35	0.83	0.83	0.87	0.49	0.60	0.68	2.64	***0.37***	***0.20***
Telecom	0.35	0.37	0.42	0.45	0.36	0.46	0.45	0.52	0.74	***0.66***
Utilities	0.35	0.27	0.30	0.32	0.27	0.26	0.42	0.23	0.41	0.51
Regime 2										
Energy	3.27	***0.65***	***0.62***	***0.47***	***0.41***	***0.36***	***0.58***	***0.37***	***0.25***	***0.52***
Materials	2.27	3.75	***0.83***	***0.73***	***0.48***	***0.32***	***0.73***	***0.51***	***0.38***	***0.41***
Industrials	1.93	2.80	3.02	***0.88***	***0.51***	***0.47***	***0.84***	***0.73***	***0.57***	***0.54***
Consumer Discr	1.69	2.79	3.01	3.88	***0.39***	***0.38***	***0.78***	***0.82***	***0.70***	***0.41***
Consumer Staples	1.04	1.33	1.27	1.09	2.02	***0.66***	***0.64***	***0.14***	***0.21***	***0.58***
Healthcare	1.00	0.95	1.26	1.17	1.45	2.38	***0.58***	***0.28***	***0.35***	***0.50***
Financials	2.19	2.97	3.06	3.22	1.90	1.86	4.37	***0.59***	***0.55***	***0.57***
IT	2.52	3.68	4.72	6.04	0.72	1.62	4.56	13.83	***0.75***	***0.24***
Telecom	1.16	1.89	2.57	3.59	0.77	1.42	2.96	7.24	6.69	***0.27***
Utilities	1.24	1.06	1.23	1.07	1.09	1.01	1.57	1.17	0.93	1.73

C. Tangency portfolio weights										
MSCI Average	0.09	0.06	0.10	0.11	0.09	0.10	0.22	0.12	0.06	0.04
C1. No constraints										
Regime 1	0.24	0.02	0.33	-0.26	0.49	0.27	-0.15	0.05	-0.30	0.30
Regime 2	0.93	-0.40	-1.97	0.56	0.58	1.04	0.21	0.66	-0.49	-0.12
Static	0.49	-0.12	-0.50	0.30	0.41	0.56	-0.19	0.21	-0.18	0.03
C2. Short constraint										
Regime 1	0.29	0.00	0.00	0.00	0.41	0.18	0.00	0.04	0.00	0.09
Regime 2	0.27	0.00	0.00	0.00	0.00	0.67	0.00	0.05	0.00	0.00
Static	0.32	0.00	0.00	0.00	0.14	0.51	0.00	0.04	0.00	0.00
C3. Benchmark Constraint										
Regime 1	0.19	-0.04	0.16	0.01	0.19	0.20	0.12	0.07	-0.04	0.14
Regime 2	0.19	-0.04	0.00	0.01	0.19	0.20	0.15	0.20	-0.04	0.14
Static	0.19	-0.04	0.00	0.14	0.19	0.20	0.12	0.10	-0.04	0.14

We report the regime-dependent means and covariances of excess returns implied by the estimates of the Regime-Switching Sector Model in Table 4. Panel A and B report the regime-dependent excess return means and covariances, where we list correlations in the upper-right triangular matrix. All numbers are listed in percentages, and are annualized. Panel C reports the mean variance efficient (MVE) (tangency) portfolios, computed using an interest rate of 6.67%, which is the average 1-month T-bill rate over the sample. The MSCI Average denotes the average MSCI world index weight of each sector across the sample. The short constraint requires weights to be positive, whereas the benchmark constraint does not allow the optimal sector weights to deviate from their MSCI average weights by more than 10%.

Table 8 Performance of Regional Allocation Portfolios

A. In-sample performance 1975 ~ 2000

	No Constraints			Short Constraint			Benchmark Constraint		
	World	Static	RS-Dynamic	World	Static	RS-Dynamic	World	Static	RS-Dynamic
Mean return (%)	13.79	16.30	19.31	13.79	16.03	16.91	13.79	15.43	15.87
Standard deviation (%)	13.84	13.58	16.51	13.84	13.46	13.79	13.84	13.29	13.47
Sharpe ratio	0.51	0.71	0.77	0.51	0.69	0.74	0.51	0.66	0.68

B. Out-of-sample performance 2001 ~ 2010

	No Constraints			Short Constraint			Benchmark Constraint		
	World	Static	RS-Dynamic	World	Static	RS-Dynamic	World	Static	RS-Dynamic
Mean return (%)	3.54	4.55	9.14	3.54	5.36	5.95	3.54	3.42	4.57
Standard deviation (%)	16.68	17.55	21.78	16.68	17.83	17.40	16.68	16.67	16.19
Sharpe ratio	0.07	0.13	0.31	0.07	0.17	0.21	0.07	0.07	0.14

We present the mean, standard deviation and Sharpe ratio of both in-sample and out-of-sample returns following the Regime-Switching Region Model and a static non-regime dependent strategy. All returns are annualized and are reported in percentages.

Table 9 Performance of Sector Allocation Portfolios

A. In-sample performance 1995 ~ 2004

	No Constraints			**Short Constraint**			**Benchmark Constraint**		
	World	Static	RS-Dynamic	World	Static	RS-Dynamic	World	Static	RS-Dynamic
Mean return (%)	7.29	14.61	18.50	7.29	11.27	10.74	7.29	9.81	10.12
Standard deviation (%)	14.39	13.06	20.34	14.39	11.65	11.66	14.39	11.83	12.45
Sharpe ratio	0.24	0.83	0.72	0.24	0.64	0.60	0.24	0.51	0.51

B. Out-of-sample performance 2005 ~ 2010

	No Constraints			**Short Constraint**			**Benchmark Constraint**		
	World	Static	RS-Dynamic	World	Static	RS-Dynamic	World	Static	RS-Dynamic
Mean return (%)	1.59	6.93	14.47	1.59	4.58	5.90	1.59	3.14	3.81
Standard deviation (%)	15.54	11.17	13.49	15.54	12.12	12.33	15.54	13.31	13.41
Sharpe ratio	-0.07	0.39	0.88	-0.07	0.16	0.27	-0.07	0.04	0.09

We present the mean, standard deviation and Sharpe ratio of both in-sample and out-of-sample returns following the Regime-Switching Sector Model and a static non-regime dependent strategy. All returns are annualized and are reported in percentages.

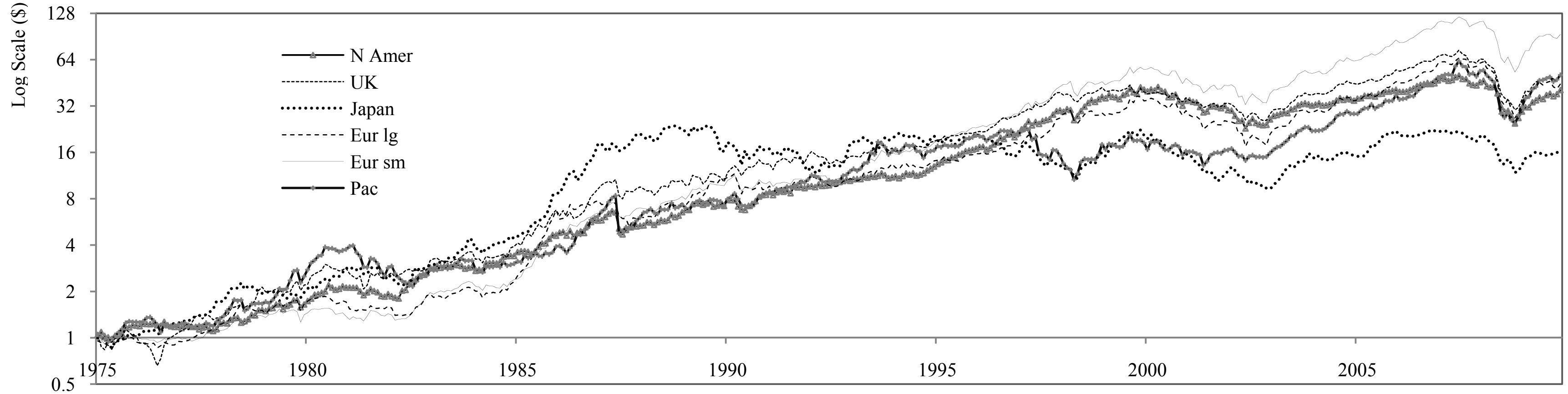
Table 10 Comparison of Cross-Region and Cross-Sector Portfolios Performance 1995 ~ 2009

Year	World		Region-Static				Region-RS				Sector-Static				Sector-RS			
	Mean	Stdev	Mean	Stdev	Beta	SR	Mean	Stdev	Beta	SR	Mean	Stdev	Beta	SR	Mean	Stdev	Beta	SR
1995	1.65	2.52	2.63	2.06	0.64	3.64	2.18	2.20	0.74	2.71	2.79	1.63	0.38	4.98	2.47	2.40	0.33	2.89
1996	1.12	2.36	1.74	2.14	0.73	2.11	1.67	1.90	0.70	2.26	2.60	3.05	1.19	2.47	2.28	2.66	0.84	2.41
1997	1.34	4.13	2.75	3.92	0.92	2.06	3.99	5.31	0.94	2.33	3.31	5.14	0.91	1.94	3.26	5.00	0.87	1.97
1998	2.02	5.67	2.62	6.04	1.03	1.27	2.34	5.75	0.91	1.17	2.65	5.06	0.62	1.54	5.17	8.97	0.77	1.84
1999	1.96	3.52	1.78	3.49	0.95	1.40	1.95	3.84	1.06	1.42	0.02	4.14	0.31	-0.31	-0.38	9.11	0.17	-0.29
2000	-1.07	4.17	-0.62	4.10	0.94	-0.93	0.64	5.25	0.78	0.10	1.28	3.70	0.18	0.75	2.00	6.36	0.39	0.83
2001	-1.37	5.25	-1.37	5.45	1.03	-1.06	-0.18	7.12	1.25	-0.24	-0.44	2.68	0.34	-0.97	-0.16	4.96	0.33	-0.33
2002	-1.65	5.59	-1.61	6.48	1.15	-0.93	-0.94	9.26	1.55	-0.40	-1.32	3.98	0.48	-1.27	-0.87	7.00	0.54	-0.50
2003	2.51	3.56	2.56	4.26	1.17	2.01	3.05	6.46	1.58	1.59	0.82	2.77	0.37	0.92	0.63	4.88	0.02	0.39
2004	1.21	2.35	1.36	2.73	1.13	1.61	1.57	2.92	1.19	1.75	0.48	2.31	0.30	0.58	1.03	1.67	0.40	1.93
2005	0.82	2.37	0.88	2.46	1.02	0.89	0.90	2.68	1.12	0.85	1.42	3.14	0.96	1.29	2.00	2.35	0.70	2.59
2006	1.60	2.09	1.94	2.11	0.99	2.54	2.24	2.38	1.13	2.68	0.49	2.90	0.74	0.12	0.88	1.94	0.58	0.87
2007	0.80	2.73	0.73	2.76	0.98	0.45	1.14	3.05	1.09	0.87	0.81	2.34	0.52	0.63	1.64	1.85	0.45	2.35
2008	-3.98	6.89	-3.78	6.95	1.00	-1.95	-4.99	7.74	1.07	-2.30	-1.22	4.56	0.50	-1.03	-0.08	6.72	0.16	-0.11
2009	2.47	6.78	2.57	7.06	1.04	1.26	4.15	7.91	1.13	1.82	1.59	3.15	0.19	1.75	1.74	5.12	-0.02	1.18
Average	0.63	4.50	0.94	4.71	1.02	0.48	1.32	5.65	1.12	0.63	1.02	3.64	0.46	0.70	1.44	5.34	0.36	0.75

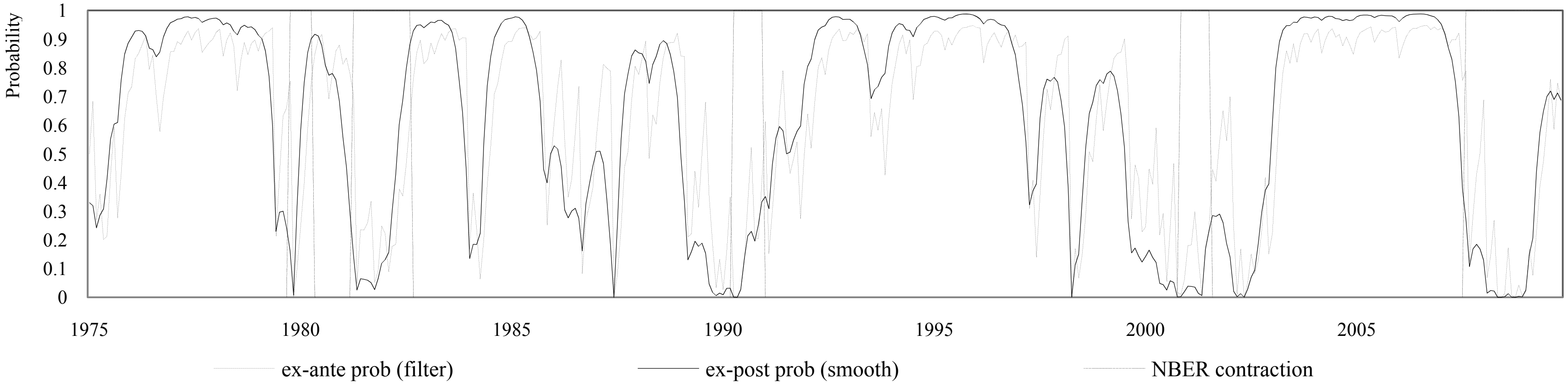
We compare the mean, standard deviation, beta and Sharpe ratio generated from static and regime switching regional allocations against their counterparts in the sector allocations. All returns and standard deviations are reported monthly and are reported in percentages.

Figure 1 Cumulated Returns and Ex Ante and Smoothed Probabilities of the Region Model 1975 ~ 2010

A. Cumulated Returns of $1.00 Invested May 1975



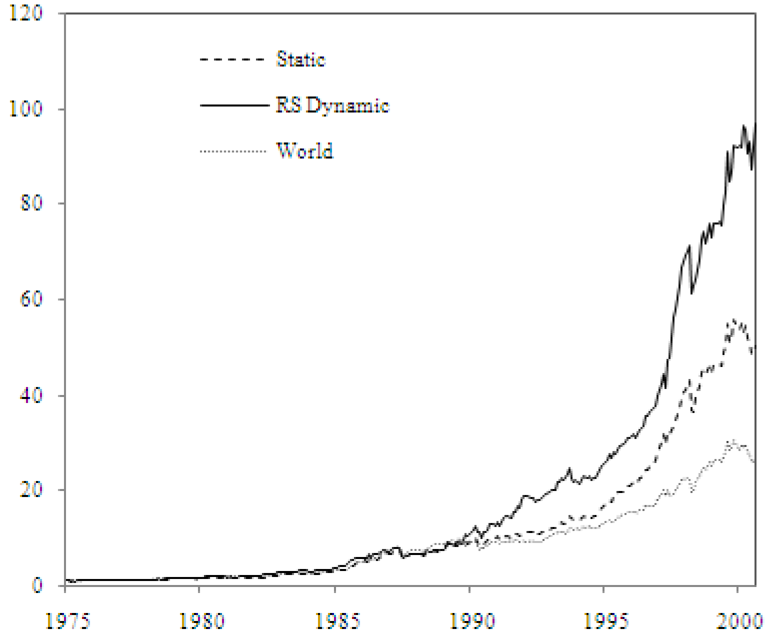
B. Ex Ante and Smoothed Probabilities of Being in (the Normal) Regime 1



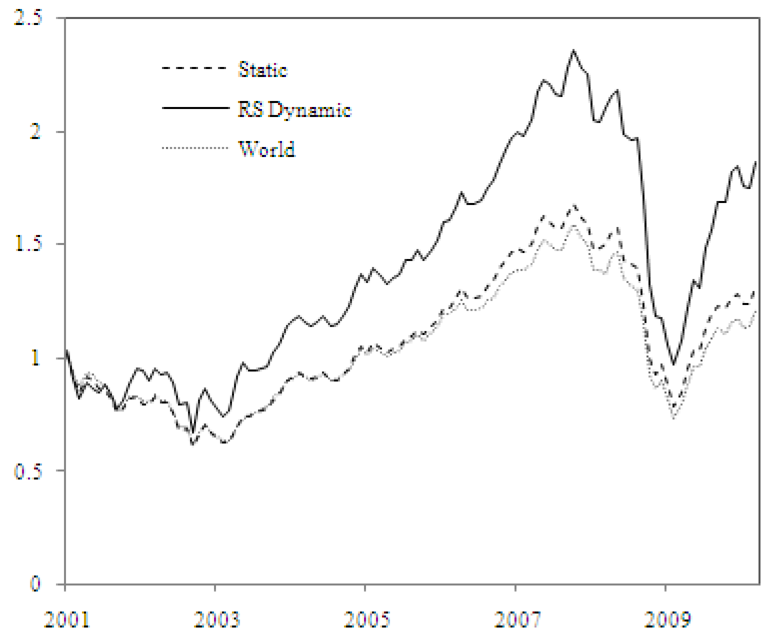
The top plot shows the accumulated total returns of $1 at May 1975 until March 2010 of each of the geographic regions. The bottom plot shows the ex ante probabilities $p(s_t = 1|I_{t-1})$ and the smoothed probabilities $p(s_t = 1|I_T)$ of being in the first regime, where the first regime is the world low variance regime.

Figure 2 In and Out-of-Sample Wealth for the Regional Allocation Model – No Constraints

A. In-Sample Regional Allocation Model 1975 ~ 2000



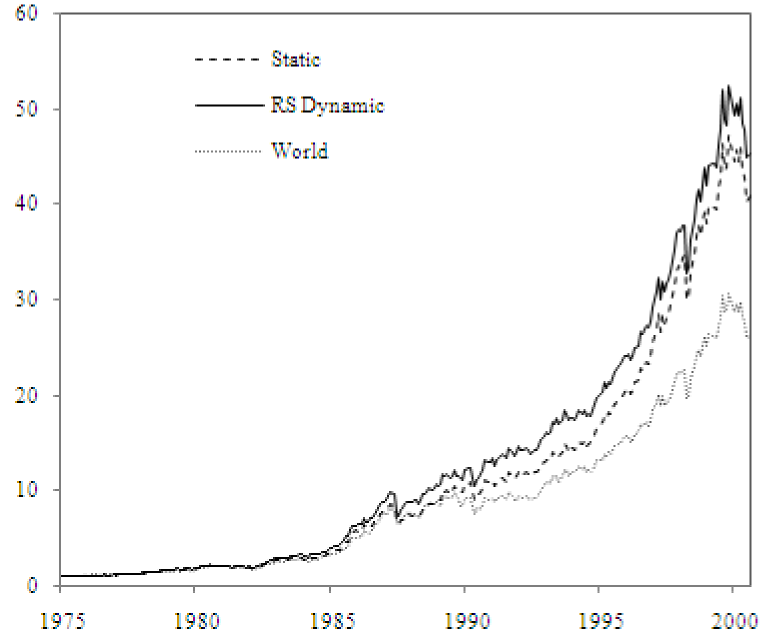
B. Out-of-Sample Regional Allocation Model 2000 ~ 2010



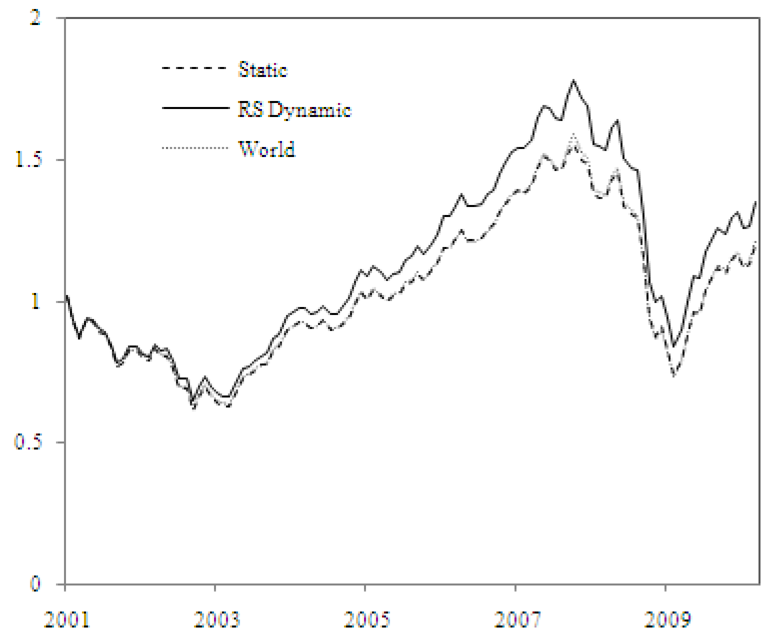
The top plot shows the in-sample wealth for the value of $1 at May 1975 for the Regime-Switching Regional Allocation Model with no constraint, contrasted with a static mean-variance strategy, and the returns for the world portfolio. The bottom plot shows the out-of-sample wealth for the value of $1 at Jan 2001 for the Regime-Switching Regional Allocation Model with no constraint.

Figure 3 In and Out-of-Sample Wealth for the Regional Allocation Model - Benchmark Constraint

A. In-Sample Regional Allocation Model 1975 ~ 2000



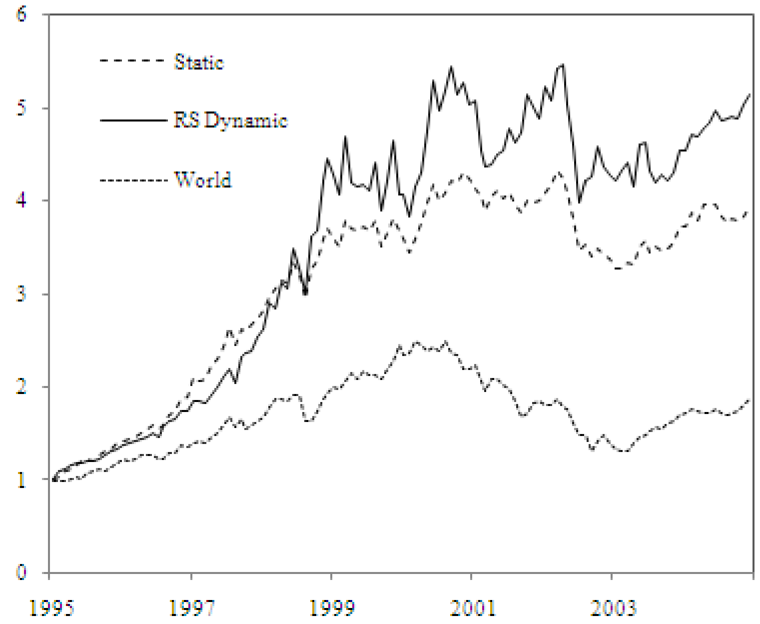
B. Out-of-Sample Regional Allocation Model 2000 ~ 2010



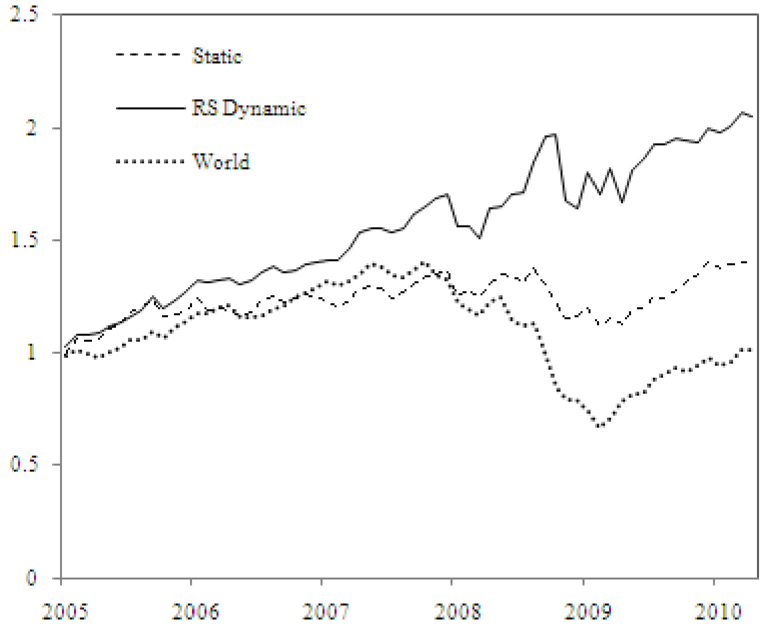
The top plot shows the in-sample wealth for the value of $1 at May 1975 for the Regime-Switching Regional Allocation Model with the benchmark constraint, contrasted with a static mean-variance strategy, and the returns for the world portfolio. The bottom plot shows the out-of-sample wealth for the value of $1 at Jan 2001 for the Regime-Switching Regional Allocation Model with the benchmark constraint.

Figure 4 In and Out-of-Sample Wealth for the Sector Allocation Model - No Constraints

A. In-Sample Sector Allocation Model 1995 ~ 2004



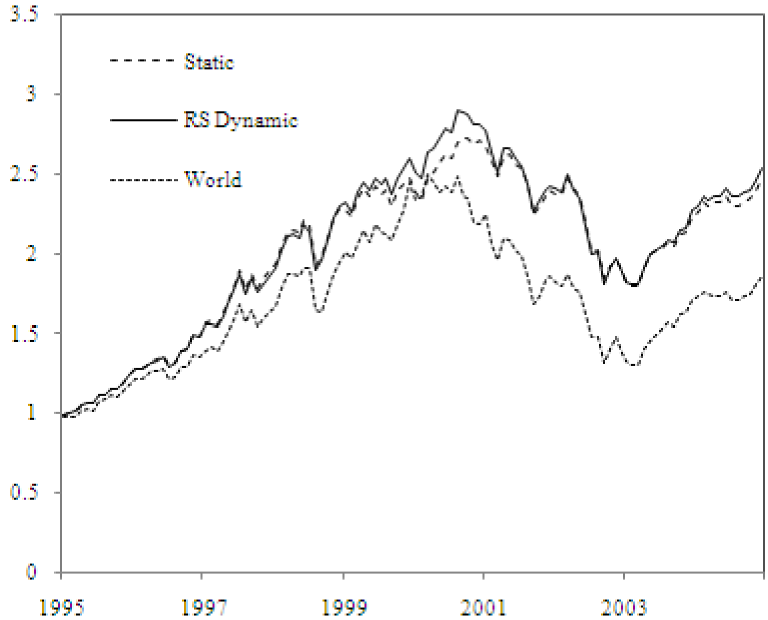
B. Out-of-Sample Sector Allocation Model 2005 ~ 2010



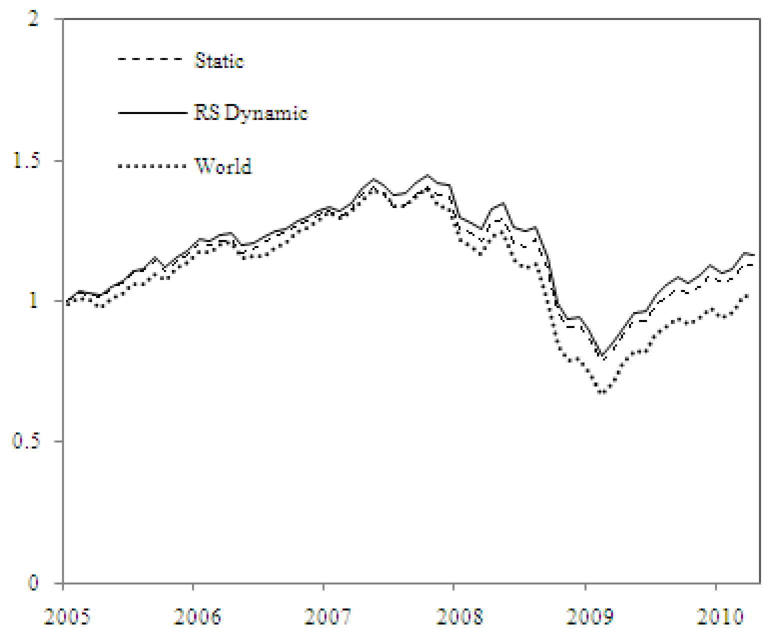
The top plot shows the in-sample wealth for the value of $1 at January 1995 for the Regime-Switching Sectoral Allocation Model with no constraint, contrasted with a static mean-variance strategy, and the returns for the world portfolio. The bottom plot shows the out-of-sample wealth for the value of $1 at Jan 2005 for the Regime-Switching Sectoral Allocation Model with no constraint.

Figure 5 In and Out-of-Sample Wealth for the Sector Allocation Model - Benchmark Constraint

A. In-Sample Sector Allocation Model 1995 ~ 2004

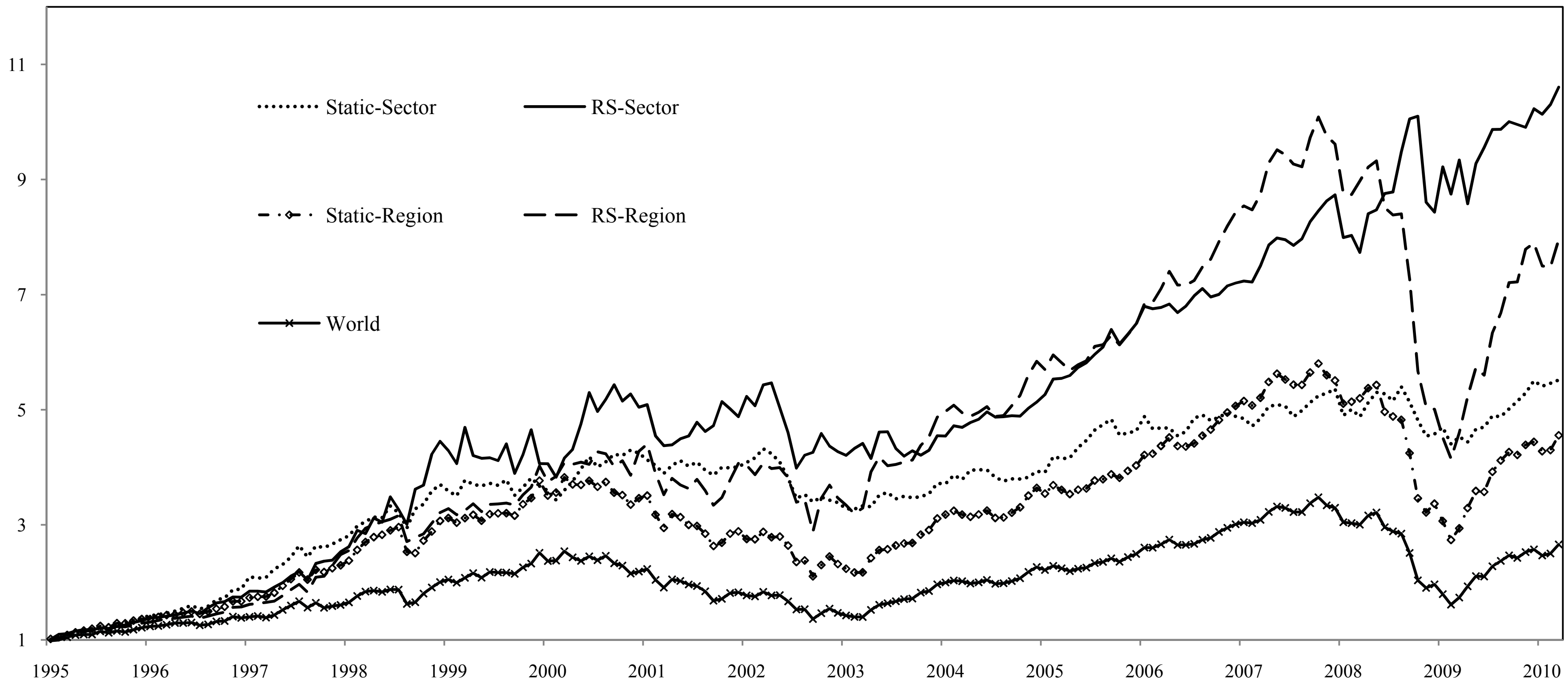


B. Out-of-Sample Sector Allocation Model 2005 ~ 2010



The top plot shows the in-sample wealth for the value of $1 at January 1995 for the Regime-Switching Sectoral Allocation Model with the benchmark constraint, contrasted with a static mean-variance strategy, and the returns for the world portfolio. The bottom plot shows the out-of-sample wealth for the value of $1 at Jan 2005 for the Regime-Switching Sectoral Allocation Model with the benchmark constraint.

Figure 6 Wealth Accumulation of $1 of the Regime Switching and Static Regional and Sector Allocation, from 1995 to 2010



The plot shows the accumulated total returns of $1 at January 1995 until March 2010 of the regime dependent and static regional and sector asset allocation with no constraints.